10.3 Operations with Radical Expressions

Review $3x + 4x = 7x$

How can you add $3\sqrt{5} + 4\sqrt{5}$?

* Like radicals - have the same radicals & can be combined by $+$ & $-$

For example, you can use the distributive property

$$3\sqrt{5} + 4\sqrt{5}$$

$$(3+4)\sqrt{5}$$

$$7\sqrt{5}$$

* Unlike radicals - have different numbers under the radical & **CAN NOT** be combined by $+$ & $-$

However make sure they are completely simplified radicals 1st

$\Longrightarrow$

Example: $\sqrt{12} - \sqrt{3}$

*You need to simplify $\sqrt{12}$ first

$\sqrt{4}\sqrt{3} - \sqrt{3}$

$2\sqrt{3} - \sqrt{3}$

$(2-1)\sqrt{3}$

$1\sqrt{3}$ or $\sqrt{3}$

10.3 Operations with Radical Expressions

* Review Problem 1 on pg. 626

* Got it #1)

A) $7\sqrt{2} - 8\sqrt{2} = -1\sqrt{2}$

B) $5\sqrt{5} + 2\sqrt{5} = 7\sqrt{5}$

* Review Problem 2 on pg. 627

* Got it #2)

A) $4\sqrt{7} + 2\sqrt{28}$

$4\sqrt{7} + 2\sqrt{4}\cdot\sqrt{7}$

$4\sqrt{7} + 2\cdot 2\sqrt{7}$

$4\sqrt{7} + 4\sqrt{7}$

$8\sqrt{7}$

B) $5\sqrt{32} - 4\sqrt{18}$

$5\sqrt{16}\cdot\sqrt{2} - 4\sqrt{9}\cdot\sqrt{2}$

$5\cdot 4\sqrt{2} - 4\cdot 3\sqrt{2}$

$20\sqrt{2} - 12\sqrt{2}$

$8\sqrt{2}$

C) No, if they are unlike & have no common factors other than 1, even if they can be simplified; they still will not be like

* Review Problem 3 on pg. 627

* Got it #3

A) $\sqrt{2}(\sqrt{6}+5)$

$\sqrt{2}\cdot\sqrt{6} + \sqrt{2}\cdot 5$

$\sqrt{12} + 5\sqrt{2}$

$\sqrt{4}\cdot\sqrt{3}$

$2\sqrt{3} + 5\sqrt{2}$

B) $(\sqrt{11}-2)^2$

$(\sqrt{11}-2)(\sqrt{11}-2)$

F O I L

$11 - 2\sqrt{11} - 2\sqrt{11} + 4$

$-4\sqrt{11} + 15$

OR

$15 - 4\sqrt{11}$

C) $(\sqrt{6}-2\sqrt{3})(4\sqrt{3}+3\sqrt{6})$

F O I L

$4\sqrt{18} + 3\sqrt{36} - 8\sqrt{9} - 6\sqrt{18}$

$4\sqrt{9}\cdot\sqrt{2} + 3\cdot 6 - (8\cdot 3) - 6\sqrt{9}\cdot\sqrt{2}$

$4\cdot 3\cdot\sqrt{2} + 18 - 24 - 6\cdot 3\sqrt{2}$

$12\sqrt{2} + 18 - 24 - 18\sqrt{2}$

$-6 - 6\sqrt{2}$

OR

$-6\sqrt{2} - 6$

<u>conjugates</u> - are the sum & difference of the same two terms ex. $\sqrt{5}+\sqrt{2}$ & $\sqrt{5}-\sqrt{2}$

* The product of conjugates is a Difference of Square which has no radical

* Review Problem 4 on pg. 628

* (Got it #4)

$$\frac{-3}{\sqrt{10}+\sqrt{5}} \cdot \frac{\sqrt{10}-\sqrt{5}}{\sqrt{10}-\sqrt{5}} = \frac{-3(\sqrt{10}-\sqrt{5})}{10-5} = \frac{-3\sqrt{10}+3\sqrt{5}}{5}$$

#38-42 even challenge

10.3 pg. 629 #8 -34 even #46-54 even

8) $\frac{1}{\sqrt{3}-1} \cdot \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{\sqrt{3}+1}{3-1} = \frac{\sqrt{3}+1}{2}$

The mistake is $\sqrt{3} \cdot \sqrt{3} \neq 9$

$\sqrt{3} \cdot \sqrt{3} = \sqrt{9} = 3$

10) $12\sqrt{5} - 3\sqrt{5} = 9\sqrt{5}$

12) $4\sqrt{2} - 7\sqrt{2} = -3\sqrt{2}$

14) $4\sqrt{128} + 5\sqrt{18}$

$4\sqrt{16}\sqrt{8} + 5\sqrt{9} \cdot \sqrt{2}$

$4 \cdot 4\sqrt{4} \cdot \sqrt{2} + 5 \cdot 3\sqrt{2}$

$16 \cdot 2\sqrt{2} + 15\sqrt{2}$

$32\sqrt{2} + 15\sqrt{2}$

$47\sqrt{2}$

16) $\sqrt{28} - 5\sqrt{7}$

$\sqrt{4} \cdot \sqrt{7} - 5\sqrt{7}$

$2\sqrt{7} - 5\sqrt{7}$

$-3\sqrt{7}$

18) $3\sqrt{3} - 2\sqrt{12}$

$3\sqrt{3} - 2 \cdot 2 \cdot \sqrt{3}$

$3\sqrt{3} - 4\sqrt{3}$

$-\sqrt{3}$

20) $5\sqrt{8} + 2\sqrt{72}$

$5\sqrt{4} \cdot \sqrt{2} + 2\sqrt{36}\sqrt{2}$

$5 \cdot 2\sqrt{2} + 2 \cdot 6\sqrt{2}$

$10\sqrt{2} + 12\sqrt{2}$

$22\sqrt{2}$

22) $\sqrt{5}(\sqrt{15} - 3)$

$\sqrt{75} - 3\sqrt{5}$

$\sqrt{25}\sqrt{3} - 3\sqrt{5}$

$5\sqrt{3} - 3\sqrt{5}$

24) $-\sqrt{12}(4 - 2\sqrt{3})$

$-4\sqrt{12} + 2\sqrt{36}$

$-4\sqrt{4}\sqrt{3} + 2 \cdot 6$

$-4 \cdot 2\sqrt{3} + 12$

$-8\sqrt{3} + 12$

26) $(3\sqrt{11}+\sqrt{7})^2$

$(3\sqrt{11}+\sqrt{7})(3\sqrt{11}+\sqrt{7})$

F O I L

$9\cdot 11 + 3\sqrt{77} + 3\sqrt{77} + 7$

$99 + 6\sqrt{77} + 7$

$106 + 6\sqrt{77}$

28) $(\sqrt{6}+\sqrt{3})(\sqrt{2}-2)$

F O I L

$\sqrt{12} - 2\sqrt{6} + \sqrt{6} - 2\sqrt{3}$

$\sqrt{4}\cdot\sqrt{3} - 1\sqrt{6} - 2\sqrt{3}$

$2\sqrt{3} - \sqrt{6} - 2\sqrt{3}$

$-\sqrt{6}$

30) $\frac{5}{\sqrt{2}-1} \circ \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{5\sqrt{2}+5}{2-1} = 5\sqrt{2}+5$

32) $\frac{-2}{\sqrt{6}+\sqrt{11}} \circ \frac{\sqrt{6}-\sqrt{11}}{\sqrt{6}-\sqrt{11}} = \frac{-2\sqrt{6}+2\sqrt{11}}{6-11} = \frac{-2\sqrt{6}+2\sqrt{11}}{-5}$

34) $\frac{-1}{2-2\sqrt{3}} \circ \frac{2+2\sqrt{3}}{2+2\sqrt{3}} = \frac{-2+2\sqrt{3}}{4-4\cdot 3} = \frac{-2-2\sqrt{3}}{4-12} = \frac{-2-2\sqrt{3}}{-8}$

$\frac{-2}{-8} + \frac{-2\sqrt{3}}{-8} = \frac{1}{4} + \frac{\sqrt{3}}{4}$ or $\frac{1+\sqrt{3}}{4}$

31) $\frac{3}{\sqrt{7}-\sqrt{3}} \circ \frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}} = \frac{3(\sqrt{7}+\sqrt{3})}{7-3}$

$\frac{3\sqrt{7}+3\sqrt{3}}{4}$

33) $\frac{\sqrt{5}}{2-\sqrt{5}} \circ \frac{2+\sqrt{5}}{2+\sqrt{5}} = \frac{2\sqrt{5}+5}{4-5} = \frac{2\sqrt{5}+5}{-1}$

$-2\sqrt{5}-5$

35) $\frac{7}{\sqrt{5}+\sqrt{13}} \circ \frac{\sqrt{5}-\sqrt{13}}{\sqrt{5}-\sqrt{13}} = \frac{7\sqrt{5}-7\sqrt{13}}{5-13} =$

$\frac{7\sqrt{5}-7\sqrt{13}}{-8}$ or $\frac{-7\sqrt{5}+7\sqrt{13}}{8}$

46) $\sqrt{6} + \sqrt{24}$
$\sqrt{6} + \sqrt{4}\sqrt{6}$
$\sqrt{6} + 2\sqrt{6}$
$3\sqrt{6}$

The student did not take the square root of 4.

48) $3\sqrt{2}(2+\sqrt{6})$
$6\sqrt{2} + 3\sqrt{12}$
$6\sqrt{2} + 3\sqrt{4}\sqrt{3}$
$6\sqrt{2} + 3\cdot 2\sqrt{3}$
$6\sqrt{2} + 6\sqrt{3}$

54) $4\sqrt{50} - 7\sqrt{18}$
$4\sqrt{25}\sqrt{2} - 7\sqrt{9}\cdot\sqrt{2}$
$4\cdot 5\sqrt{2} - 7\cdot 3\sqrt{2}$
$20\sqrt{2} - 21\sqrt{2}$
$-\sqrt{2}$

50) $(\sqrt{3}+\sqrt{5})^2$
$(\sqrt{3}+\sqrt{5})(\sqrt{3}+\sqrt{5})$
$\sqrt{9} + \sqrt{15} + \sqrt{15} + \sqrt{25}$
$3 + 2\sqrt{15} + 5$
$8 + 2\sqrt{15}$

Challenge

38) $\frac{5\sqrt{2}}{\sqrt{2}-1} = \frac{x}{\sqrt{2}}$

$5\sqrt{4} = x\sqrt{2} - x$
$5\cdot 2 = x\sqrt{2} - x$
$10 = x\sqrt{2} - x$
$\frac{10 = x(\sqrt{2}-1)}{\sqrt{2}-1}$

$x = \frac{10}{\sqrt{2}-1} \cdot \frac{\sqrt{2}+1}{\sqrt{2}+1}$

$x = \frac{10(\sqrt{2}+1)}{2-1} = \frac{10(\sqrt{2}+1)}{1}$

$x = 10(\sqrt{2}+1)$

52) $(\sqrt{7}+\sqrt{8})(\sqrt{7}+\sqrt{8})$
$\sqrt{49} + \sqrt{56} + \sqrt{56} + \sqrt{64}$
$7 + \sqrt{4}\sqrt{14} + \sqrt{4}\sqrt{14} + 8$
$7 + 2\sqrt{14} + 2\sqrt{14} + 8$
$7 + 4\sqrt{14} + 8$
$15 + 4\sqrt{14}$

40) $\frac{\sqrt{2}-1}{\sqrt{2}+1} = \frac{x}{2}$

$2\sqrt{2} - 2 = x\sqrt{2} + x$

$\frac{2\sqrt{2} - 2 = x(\sqrt{2}+1)}{\sqrt{2}+1}$

$\frac{2\sqrt{2} - 2}{\sqrt{2}+1} \cdot \frac{\sqrt{2}-1}{\sqrt{2}-1}$

$\frac{(2\sqrt{2}-2)(\sqrt{2}-1)}{2-1} = \frac{\quad}{1}$

F O I L

$2\sqrt{4} \quad -2\sqrt{2} \quad -2\sqrt{2} + 2$

$2 \cdot 2 - 4\sqrt{2} + 2$

$4 - 4\sqrt{2} + 2$

$6 - 4\sqrt{2}$

Common denominator

$2\left(\frac{\sqrt{15}}{15}\right) + \left(\frac{\sqrt{5}}{10}\right)3$

$\frac{2\sqrt{15}}{30} + \frac{3\sqrt{5}}{30}$

$\frac{2\sqrt{15} + 3\sqrt{5}}{30}$

42) $\frac{4\sqrt{15}}{1+\sqrt{3}} = \frac{1+\sqrt{3}}{x}$

$4x\sqrt{15} = (1+\sqrt{3})(1+\sqrt{3})$

$4x\sqrt{15} = \quad$ F O I L

$4x\sqrt{15} = 1 + \sqrt{3} + \sqrt{3} + \sqrt{9}$

$4x\sqrt{15} = 1 + 2\sqrt{3} + 3$

$\frac{4x\sqrt{15} = 4 + 2\sqrt{3}}{4\sqrt{15}}$

$x = \frac{4 + 2\sqrt{3}}{4\sqrt{15}}$

$\frac{4+2\sqrt{3}}{4\sqrt{15}} \cdot \frac{4\sqrt{15}}{4\sqrt{15}}$

$\frac{4\sqrt{15}(4+2\sqrt{3})}{16 \cdot 15}$

$\frac{16\sqrt{15} + 8\sqrt{45}}{240}$

$16\sqrt{15} + 8\sqrt{5} \cdot \sqrt{9}$

$16\sqrt{15} + 8 \cdot \sqrt{5} \cdot 3$

$\frac{16\sqrt{15} + 24\sqrt{5}}{240}$

$\frac{\sqrt{15}}{15} + \frac{\sqrt{5}}{10}$