8.6 Factoring $ax^2 + bx + c$

Examples:

① $2x^2 + 5x + 3$

$2x^2 + 2x + 3x + 3$

$(2x^2 + 2x) + (3x + 3)$
$2x(x+1) + 3(x+1)$

$(2x + 3)(x + 1)$

Factors of AC (6)	Sum of B (5)
1, 6	7
2, 3	5

$\rightarrow$ F O I L
$2x^2 + 2x + 3x + 3$

$2x^2 + 5x + 3 \checkmark$

② $6m^2 + 7m - 5$

$6m^2 - 3m + 10m - 5$

$(6m^2 - 3m) + (10m - 5)$

$3m(2m-1) + 5(2m-1)$

$(3m + 5)(2m - 1)$

Factors of AC (-30)	sum of B (7)
1, -30	-29
-1, 30	29
2, -15	-13
-2, 15	13
-3, 10	7

* Check w/ FOIL

③ $8x^2 - 36x - 20$

* Always Factor out a GCF 1st **

$4(2x^2 - 9x - 5)$

Factors of AC (-10)	Sum of B (-9)
$1, -10$	-9

$4(2x^2 + 1x - 10x - 5)$

$4((2x^2 + 1x) + (-10x - 5))$

$4(x(2x+1) + -5(2x+1))$

$4(x-5)(2x+1)$

CHECK $4(x-5)(2x+1)$

F O I L

$4(x^2 + 1x - 10x - 5)$

$4(x^2 - 9x - 5)$

$4x^2 - 36x - 20$ ✓

→ Got it #1 B) The factors are both negative

→ How can you find the dimesions of the rectangle that has an Area of a quadratic trinomial? (Got it #3 ?)
→ Factor the rectangle's area as the product of 2 binomials, one of which is the width. The other must be the length since $A = \ell \cdot w$.

8-6 Practice

Form G

Factoring $ax^2 + bx + c$

Factor each expression.

1. $2w^2 + 13w + 15$

2. $3d^2 + 20d + 12$

3. $4n^2 + 62n - 32$

4. $3p^2 - 7p - 40$

5. $6r^2 - 10r - 24$

6. $5z^2 - 17z + 14$

7. $14k^2 - 67k + 63$

8. $2m^2 - m - 15$

9. $3x^2 + 9x - 84$

10. $4y^2 + 26y + 30$

11. $5t^2 - 24t - 5$

12. $7c^2 - 2c - 9$

13. $8k^2 - 42k + 27$

14. $6g^2 - 2g - 20$

15. $2c^2 - 23c + 11$

16. The area of a rectangular computer screen is $4x^2 + 20x + 16$. The width of the screen is $2x + 8$. What is the length of the screen?

17. The area of a rectangular granite countertop is $12x^2 + 10x - 12$. The width of the countertop is $2x + 3$. What is the length of the countertop?

18. The area of a rectangular book cover is $4x^2 - 6x - 40$. The width of the book cover is $2x - 8$. What is the length of the book cover?

19. The area of a rectangular parking lot is $21x^2 - 44x + 15$. The width of the parking lot is $3x - 5$. What is the length of the parking lot?

Factor each expression completely.

20. $6x^2 - 10x - 4$

21. $6d^2 + 21d + 15$

22. $8n^2 + 68n + 84$

23. $20p^2 - 115p - 30$

24. $15r^2 + 141r - 90$

25. $12z^2 - 14z + 4$

26. $20k^2 + 110k + 120$

27. $9m^2 - 66m + 21$

28. $40x^2 - 136x - 96$

29. $42y^2 + 28y - 14$

30. $8t^2 - 16t - 90$

31. $24c^2 + 96c + 90$

8-6 Practice   Form G

Factoring $ax^2 + bx + c$

(continued)

Open-Ended Find two different values that complete each expression so that the trinomial can be factored into the product of two binomials. Factor your trinomials.

32. $4x^2 + \boxed{}\, x + 12$

33. $6t^2 - \boxed{}\, t - 4$

34. $9m^2 - \boxed{}\, m + 8$

35. $8n^2 + \boxed{}\, n - 10$

36. $12v^2 - \boxed{}\, v + 15$

37. $5w^2 - \boxed{}\, w - 24$

38. Error Analysis Describe and correct the error made in factoring the expression at the right.

$$(6x^2 + 3x - 9) = 3(2x^2 + x - 3)$$
$$= 3(2x^2 - 3x + 2x - 3)$$
$$= 3(2x^2 - 3x + (2x - 3)$$
$$= 3[\, x\,(2x - 3) + 1\,(2x - 3)\,]$$
$$= 3(x + 1)\,(2x - 3)$$

39. A parallelogram has an area of $4x^2 + 7x - 15$. The base of the parallelogram is $x + 3$. What is the height of the parallelogram?
 a. Write the formula for the area of a parallelogram.

 b. Writing Explain how factoring the trinomial helps you solve the problem.

40. A rectangular window pane has an area of $15x^2 - 19x + 6$. The width of the window pane is $3x - 2$. What is the length of the window pane?

Factor each expression completely.

41. $28y^2 + 43y - 48$

42. $16z^2 - 54z + 35$

43. $27n^2 - 54n + 15$

44. $36p^2 + 63p + 20$

45. $28r^2 - 20r - 33$

46. $30z^2 - 53z + 12$

47. $32x^3 + 28x^2 + 5x$

48. $25p^2 + 20pq - 12q^2$

49. $72g^2h - 43gh + 6h$

8-6 **Practice** Form G

Factoring $ax^2 + bx + c$

(handwritten right margin: EVENS #1-30 all SKIP 2 & 38 & 39)

Factor each expression.

1. $2w^2 + 13w + 15$
$(2w + 3)(w + 5)$

2. $3d^2 + 20d + 12$
$(3d + 2)(d + 6)$

3. $4n^2 + 62n - 32$
$2(2n - 1)(n + 16)$

4. $3p^2 - 7p - 40$
$(3p + 8)(p - 5)$

5. $6r^2 - 10r - 24$
$2(3r + 4)(r - 3)$

6. $5z^2 - 17z + 14$
$(5z - 7)(z - 2)$

7. $14k^2 - 67k + 63$ *(SKIP)*
$(2k - 7)(7k - 9)$

8. $2m^2 - m - 15$
$(2m + 5)(m - 3)$

9. $3x^2 + 9x - 84$
$3(x + 7)(x - 4)$

10. $4y^2 + 26y + 30$
$2(2y + 3)(y + 5)$

11. $5t^2 - 24t - 5$
$(5t + 1)(t - 5)$

12. $7c^2 - 2c - 9$
$(7c - 9)(c + 1)$

13. $8k^2 - 42k + 27$ *(SKIP)*
$(4k - 3)(2k - 9)$

14. $6g^2 - 2g - 20$
$2(3g + 5)(g - 2)$

15. $2c^2 - 23c + 11$
$(2c - 1)(c - 11)$

(handwritten: Review = 7)

16. The area of a rectangular computer screen is $4x^2 + 20x + 16$. The width of the screen is $2x + 8$. What is the length of the screen?
$2x + 2$ *(handwritten: $2(x+4)$; $4(x^2+5x+4)$; $4(x+1)(x+4)$)*

17. The area of a rectangular granite countertop is $12x^2 + 10x - 12$. The width of the countertop is $2x + 3$. What is the length of the countertop?
$6x - 4$ *(handwritten: $2(x+1)(x+4)$; $2(x+1) \cdot 2(x+4)$)*

18. The area of a rectangular book cover is $4x^2 - 6x - 40$. The width of the book cover is $2x - 8$. What is the length of the book cover?
$2x + 5$

19. The area of a rectangular parking lot is $21x^2 - 44x + 15$. The width of the parking lot is $3x - 5$. What is the length of the parking lot? *(SKIP)*
$7x - 3$

Factor each expression completely.

20. $6x^2 - 10x - 4$
$2(3x + 1)(x - 2)$

21. $6d^2 + 21d + 15$
$3(2d + 5)(d + 1)$

22. $8n^2 + 68n + 84$
$4(2n + 3)(n + 7)$

23. $20p^2 - 115p - 30$
$5(4p + 1)(p - 6)$

24. $15r^2 + 141r - 90$ *(SKIP)*
$3(5r - 3)(r + 10)$

25. $12z^2 - 14z + 4$
$2(2z - 1)(3z - 2)$

26. $20k^2 + 110k + 120$
$10(2k + 3)(k + 4)$

27. $9m^2 - 66m + 21$
$3(3m - 1)(m - 7)$

28. $40x^2 - 136x - 96$
$8(5x + 3)(x - 4)$

29. $42y^2 + 28y - 14$
$14(3y - 1)(y + 1)$

30. $8t^2 - 16t - 90$
$2(2t + 5)(2t - 9)$

31. $24c^2 + 96c + 90$
$6(2c + 5)(2c + 3)$

8-6 Practice (continued) — Form G
Factoring $ax^2 + bx + c$

Open-Ended Find two different values that complete each expression so that the trinomial can be factored into the product of two binomials. Factor your trinomials.

32. $4x^2 + \boxed{}x + 12$

Answers may vary. Sample:
19, 16: $(4x + 3)(x + 4)$;
$(4x + 4)(x + 3)$

33. $6t^2 - \boxed{}t - 4$

Answers may vary. Sample:
23, −5; $(6t + 1)(t − 4)$;
$(3t + 4)(2t − 1)$

34. $9m^2 - \boxed{}m + 8$

Answers may vary. Sample:
73, 27; $(9m − 1)(m − 8)$;
$(3m − 8)(3m − 1)$

35. $8n^2 + \boxed{}n - 10$

Answers may vary. Sample:
11, −11; $(8n − 5)(n + 2)$;
$(n − 2)(8n + 5)$

36. $12v^2 - \boxed{}v + 15$

Answers may vary. Sample:
29, 27; $(4v − 3)(3v − 5)$;
$(4v − 5)(3v − 3)$

37. $5w^2 - \boxed{}w - 24$

Answers may vary. Sample.
26, 14; $(5w + 4)(w − 6)$;
$(5w + 6)(w − 4)$

38. Error Analysis Describe and correct the error made in factoring the expression at the right.

In the second step, the student wrote −1x instead of 1x. x should be written as 3x − 2x. Answer: $3(2x + 3)(x − 1)$

$(6x^2 + 3x − 9) = 3(2x^2 + x − 3)$
$= 3(2x^2 − 3x + 2x − 3)$
$= 3(2x^2 − 3x + (2x − 3)$
$= 3[x (2x − 3) + 1 (2x − 3)]$
$= 3(x + 1) (2x − 3)$

Product of −6 / 3 and 2 / Sum of 1

39. A parallelogram has an area of $4x^2 + 7x − 15$. The base of the parallelogram is $x + 3$. What is the height of the parallelogram?

a. Write the formula for the area of a parallelogram. $A = bh$

b. Writing Explain how factoring the trinomial helps you solve the problem.

Factor to find h: $(x + 3)(4x − 5) = 4x^2 + 7x − 15$; $h = 4x − 5$

$4x^2+7x-15$
Product of 60, 5 & 12 / Sum of 7
$4x^2-5x+12x-15$
$x(4x-5)+3(4x-5)$
$(x+3)(4x-5)$

40. A rectangular window pane has an area of $15x^2 − 19x + 6$. The width of the window pane is $3x − 2$. What is the length of the window pane?

$5x − 3$

$15x^2-19x+6$

Factor each expression completely.

41. $28y^2 + 43y − 48$
$(4y − 3)(7y + 16)$

42. $16z^2 − 54z + 35$
$(8z − 7)(2z − 5)$

43. $27n^2 − 54n + 15$
$3(3n − 1)(3n − 5)$

44. $36p^2 + 63p + 20$
$(3p + 4)(12p + 5)$

45. $28r^2 − 20r − 33$
$(2r − 3)(14r + 11)$

46. $30z^2 − 53z + 12$
$(2z − 3)(15z − 4)$

47. $32x^3 + 28x^2 + 5x$
$x(4x + 1)(8x + 5)$

48. $25p^2 + 20pq − 12q^2$
$(5p − 2q)(5p + 6q)$

49. $72g^2h − 43gh + 6h$
$h(9g − 2)(8g − 3)$