10.1 Simplifying Rational Expressions

* **Rational Expression**

→ is a quotient of 2 polynomials (Ex $\frac{5}{x}$)

→ denominator cannot be simplified to zero (A rational expression is undefined when the denominator is 0. A value of a variable for which a rational expression is undefined is an excluded value.)

$\frac{5}{x}$, $x \neq 0$ (The excluded value is 0.)

$\frac{a}{b-2}$, $b \neq 2$ (The excluded value is 2.)

$\frac{5}{x-y}$, $x \neq y$ (The excluded value is y).

* Rational Expressions in simplest form means the numerator & denominator have NO common factors other than 1 or -1

Examples: $\frac{x+y}{x}$ * x is not a factor, it is a term

* already in simplest form

$\frac{2x+2y}{2x}$ * Need to factor out 2

$\frac{2(x+y)}{2x}$ * Cancel out the 2; now it is simplest form

To Simplify Rational Expressions....

1) factor the numerator & denominator
2) Reduce common factors to 1 or -1

Ex $\frac{4x^2y}{2xy^3} = \frac{2xy(2x)}{2xy(y^2)} = \frac{2x}{y^2}$, $y \neq 0$

$\frac{9x+6}{3x} = \frac{3(3x+2)}{3} = \frac{3x+2}{x}$, $x \neq 0$

$\frac{x^2+6x+9}{3x+9} = \frac{(x+3)(x+3)}{3(x+3)} = \frac{x+3}{3}$, $x \neq -3$

$\frac{a^2-9}{a^2+5a+6} = \frac{(a+3)(a-3)}{(a+3)(a+2)} = \frac{a-3}{a+2}$, $a \neq -2$ or $a \neq -3$

$\frac{a^2-1}{a-1} = \frac{(a+1)(a-1)}{(a-1)} = a+1$, $a \neq 1$

$\frac{b-7^*}{7-b^*} = \frac{-1(7-b)}{7-b} = -1$, $b \neq 7$

$\frac{a-2b}{4b-2a} = \frac{a-2b^*}{2(2b-a^*)} = \frac{-1(a-2b)}{2(2b-a)} = \frac{-1}{2}$

$b \neq \frac{a}{2}$ or $a \neq 2b$

* additive inverses (opposites) equal -1

* Got it #1

A) $\frac{21a^2}{7a^3} = \frac{3}{a}, a \neq 0$

B) $\frac{18d^2}{4d+8} = \frac{18d^2}{4(d+2)} = \frac{9d^2}{2(d+2)} = \frac{9d^2}{2d+4}, d \neq -2$

C) $\frac{2n-3}{6n-9} = \frac{2n-3}{3(2n-3)} = \frac{1}{3}, n \neq 3/2$

$6n-9=0$
$+9 \quad +9$
$\frac{6n=9}{6}$
$n=3/2$

D) $\frac{26c^3+91c}{2c^2+7} = \frac{13c(2c^2+7)}{2c^2+7} = 13c$

* Got it #2

A) $\frac{2x-8}{x^2-2x-8} = \frac{2(x-4)}{(x+2)(x-4)} = \frac{2}{(x+2)}, x \neq -2 \text{ \& } x \neq 4$

↓ Factor x^2-2x-8

$x^2-4x+2x-8$

$x(x-4)+2(x-4)$

$(x+2)(x-4)$

* Find the restricted values before simplifying!

B) $\frac{a^2-3a+2}{3a-3} = \frac{(a-1)(a-2)}{3(a-1)} = \frac{(a-2)}{3}, a \neq 1$

Factor the Numerator: a^2-3a+2

$a^2-2a-1a+2$

$(a^2-2a)+(-1a+2)$

$a(a-2)+-1(a-2)$

$(a-1)(a-2)$

C) $\frac{6z+12}{2z^2+7z+6} = \frac{6(z+2)}{(z+2)(2z+3)} = \frac{6}{2z+3}, z \neq \frac{-3}{2}$

Factor: $2z^2+7z+6$

$2z^2+3z+4z+6$

$(2z^2+3z)+(4z+6)$

$z(2z+3)+2(2z+3)$

$(z+2)(2z+3)$

$2z+3=0$

$-3 \quad -3$

$\frac{2z=-3}{2}$

$z=-\frac{3}{2}$

D) $\frac{c^2-c-6}{c^2+5c+6} = \frac{(c+2)(c-3)}{(c+2)(c+3)} = \frac{c-3}{c+3}, c \neq -2 \text{ or } -3$

Factor Numerator

c^2-c-6

$c^2-3c+2c-6$

$(c^2-3c)+(2c-6)$

$c(c-3)+2(c-3)$

$(c+2)(c-3)$

Factor Denominator

c^2+5c+6

$c^2+3c+2c+6$

$(c^2+3c)+(2c+6)$

$c(c+3)+2(c+3)$

$(c+2)(c+3)$

* Got it #3

A) $\frac{2x-5}{5-2x} = \frac{-1(2x+5)}{5-2x} = \frac{-1(5-2x)}{5-2x} = -1,\ x \neq \frac{5}{2}$

B) $\frac{y^2-16}{4-y} = \frac{(y+4)(y-4)}{4-y} = \frac{-1(4-y)(y+4)}{4-y} = -1(y+4)$

or $-y-4,\ y \neq 4$

C) $\frac{3-9d}{6d^2+d-1} = \frac{-3(-1+3d)}{(3d-1)(2d+1)} = \frac{-3}{2d+1},\ d \neq \frac{1}{3}$ or $\frac{-1}{2}$

Factor denominator

$6d^2+d-1$

$6d^2+3d-2d-1$

$(6d^2+3d)+(-2d-1)$

$3d(2d+1)+-1(2d+1)$

$(3d-1)(2d+1)$

$3d-1=0$

$+1\ +1$

$\frac{3d=1}{3}$

$d=\frac{1}{3}$

$2d+1=0$

$-1\ -1$

$\frac{2d=-1}{2}$

$d=-\frac{1}{2}$

D) $\frac{3-3z}{2z^2-2} = \frac{-3(-1+z)}{2(z^2-1)} = \frac{-3(-1+z)}{2(z+1)(z-1)} = \frac{-3}{2(z+1)} =$

$\frac{-3}{2z+2}$, $z \neq \pm 1$

* Review Problem 4 on pg. 6066

* Got it #4) Area of the square = Area of the rectangle

A) $6x+2 = 3x+1$

$s^2 = l \cdot w$

$(6x+2)^2 = l(3x+1)$

$(6x+2)(6x+2) = l(3x+1)$

$\frac{2(3x+1)2(3x+1) = l(3x+1)}{3x+1}$

$4(3x+1) = l$

$12x+4 =$ length of the rectangle

* Remember: You can cancel out factors by division NOT terms!

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5) If the denominator contains a polynomial, there may be values of the variable that make the denominator equal to zero, & division by zero is undefined

7) A) $3-x$ & $x-3$ are opposites
B) $2-y$ & $-y+2$ are not opposites

9) $\frac{4x^3}{28x^4} = \frac{1}{7x}, x \neq 0$

11) $\frac{2p-24}{4p-48} = \frac{2(p-12)}{4(p-12)} = \frac{1}{2}, p \neq 12$

13) $\frac{3x+6}{3x^2} = \frac{3(x+2)}{3x^2} = \frac{x+2}{x^2}, x \neq 0$

15) $\frac{2b-8}{b^2-16} = \frac{2(b-4)}{(b-4)(b+4)} = \frac{2}{b+4}, b \neq 4 \text{ or } -4$

17) $\frac{w^2+7w}{w^2-49} = \frac{w(w+7)}{(w+7)(w-7)} = \frac{w}{w-7}, w \neq \pm 7$

19) $\frac{m^2+7m+12}{m^2+6m+8} = \frac{(m+4)(m+3)}{(m+2)(m+4)} = \frac{m+3}{m+2}, m \neq -4 \text{ or } -2$

$m^2+7m+12$
$m^2+3m+4m+12$
$m(m+3)+4(m+3)$
$(m+4)(m+3)$

m^2+6m+8
$m^2+4m+2m+8$
$m(m+4)+2(m+4)$
$(m+2)(m+4)$

21) $\frac{b^2+8b+15}{b+5} = \frac{(b+5)(b+3)}{b+5} = b+3, b \neq -5$

$b^2+8b+15$
$b^2+3b+5b+15$
$b(b+3)+5(b+3)$

23) $\frac{5-4n}{4n-5} = \frac{-1(-5+4n)}{4n-5} = -1, n \neq 5/4$

25) $\frac{4m-8}{4-2m} = \frac{-4(-m+2)}{2(2-m)} = \frac{-4}{2} = -2, m \neq 2$

27) $\frac{v-5}{25-v^2} = \frac{v-5}{(5+v)(5-v)} = \frac{-1(-v+5)}{(5+v)(5-v)} = \frac{-1}{5+v}, v \neq \pm 5$

29) $V = l \cdot w \cdot h$

$2w^3+7w^2+5w \quad (5+2w)(w)h$

$\frac{2w^3+7w^2+5w}{2w^2+5w} = \frac{(5w+2w^2)h}{2w^2+5w}$

$2w^3+7w^2+5w$

$w(2w^2+7w+5)$

$w(2w^2+5w+2w+5)$

$w(w(2w+5)+1(2w+5)$

$w((w+1)(2w+5)$

$\frac{w(w+1)(2w+5)}{w(2w+5)} = w+1, \; w \neq -\frac{5}{2} \text{ or } 0$

* Do not really need to state this measurment. Cannot be zero or negative.

31) $\frac{2r^2+9r-5}{r^2+10r+25} = \frac{(2r-1)(r+5)}{(r+5)(r+5)} = \frac{2r-1}{r+5}, \; r \neq -5$

$2r^2+9r-5$

$2r^2+10r-1r-5$

$2r(r+5)-1(r+5)$

$(2r-1)(r+5)$

$r^2+10r+25$

$(r+5)(r+5)$

*Perfect Trinomial Square

33) $\frac{5t^2+6t-8}{3t^2+5t-2} = \frac{(5t-4)(t+2)}{(3t-1)(t+2)} = \frac{5t-4}{3t-1}, \; t \neq -2 \text{ or } \frac{1}{3}$

$5t^2+6t-8$

$5t^2+10t-4t-8$

$5t(t+2)-4(t+2)$

$(5t-4)(t+2)$

$3t^2+5t-2$

$3t^2+6t-1t-2t$

$3t(t+2)-1(t+2)$

$(3t-1)(t+2)$

35) $\frac{3z^2+12z}{z^4} = \frac{3z(z+4)}{z^4} = \frac{3(z+4)}{z^3} = \frac{3z+12}{z^3}$, $z \neq 0$

37) $\frac{4a^2-8a-5}{15-a-2a^2} = \frac{(2a+1)(2a-5)}{(3+a)(5-2a)} = \frac{-1(2a+1)(5-2a)}{(3+a)(5-2a)} =$

$\frac{-(2a+1)}{3+a}$, $a \neq -3$ or $5/2$

$4a^2-8a-5$
$4a^2-10a+2a-5$
$2a(2a-5)+1(2a-5)$
$(2a+1)(2a-5)$

$15-a-2a^2$
$15-6a+5a-2a^2$
$3(5-2a)+a(5-2a)$
$(3+a)(5-2a)$

Another way to write... all the same values

$$\frac{-(2a+1)}{3+a} = \frac{-2a-1}{3+a} = -\frac{2a+1}{a+3}$$

43) The student cancelled out terms instead of factors

$$\frac{x^2+2x}{2x} = \frac{x(x+2)}{2x} = \frac{x+2}{2}, \quad x \neq 0$$