# 10-4 Solving Radical Equations

* Radical equations - an equation that has a variable in the radicand

  Examples: $\sqrt{x} - 5 = 3$

  or

  $\sqrt{x-2} = 1$

* To solve equations with radicals, we first convert them to equations without radicals. We do this by squaring both sides of the equation.

* The Principal of Squaring:
  If an equation $a=b$ is true, then the equation $a^2=b^2$ is true.

* Review Problem 1 on pg. 633.

* Example for solving a radical equation.

$\sqrt{5x} + 7 = 10$

$-7 \quad -7$ Combine like terms

$(\sqrt{5x} = 3)^2$ Square both sides

$\frac{5x = 9}{5}$ Solve for x

$x = \frac{9}{5}$

→

$\sqrt{5x} + 7 = 10$ Check $\left(x = \frac{9}{5}\right)$

$\sqrt{5\left(\frac{9}{5}\right)} + 7 = 10$

$\sqrt{9} + 7 = 10$

$3 + 7 = 10$

$10 = 10 \checkmark$

* Got it #1) $\sqrt{x} - 5 = -2$

$+5 \quad +5$

$\left(\sqrt{x} = 3\right)^2$

$x = 9$

Check:

$\sqrt{x} - 5 = -2$

$\sqrt{9} - 5 = -2$

$3 - 5 = -2$

$-2 = -2 \checkmark$

* Review Problem 2 on pg. 634

* Got it #2) $t = 2\sqrt{\frac{l}{3.3}}$

$\frac{1 = 2\sqrt{\frac{l}{3.3}}}{2}$

$\left(\frac{1}{2} = \sqrt{\frac{l}{3.3}}\right)^2$

$3.3\left(\frac{1}{4} = \frac{l}{3.3}\right)$

$0.825 = l$

0.825 ft Rounded to 0.8 ft

* Review Problem 3 on pg. 634

* Solve w/ Radicals on Both Sides)

Example:

$$\sqrt{3x-1} = \sqrt{x+5}$$

$$(\sqrt{3x-1})^2 = (\sqrt{x+5})^2$$

$$3x-1 = x+5$$

$$-x \qquad -x$$

$$2x-1 = 5$$

$$+1 \qquad +1$$

$$\frac{2x = 6}{2}$$

$$x = 3$$

Check: $\sqrt{3 \cdot 3 - 1} = \sqrt{3+5}$

$$\sqrt{8} = \sqrt{8} \checkmark$$

* Got it #3) $\sqrt{7x-4} = \sqrt{5x+10}$

$$(\sqrt{7x-4} = \sqrt{5x+10})^2$$

$$7x-4 = 5x+10$$

$$+4 \qquad +4$$

$$7x = 5x+14$$

$$-5x \quad -5x$$

$$2x = 14$$

$$x = 7$$

Check: $\sqrt{7 \cdot 7 - 4} = \sqrt{5 \cdot 7 + 10}$

$$\sqrt{45} = \sqrt{45} \checkmark$$

* When solving equations, it is ALWAYS necessary to check each solution of the new equation. (When both sides of an equation are modified, the new equation may have extra or extranlous solutions.)

* Extraneous solutions - apparent solution that does not satisfy the original equation

* See top of pg. 635 as an example

* Review Problem 4 on pg. 635

* Got it #4 $\quad -y = \sqrt{y+6}$

$$(-y = \sqrt{y+6})^2$$

$$+y^2 = y+6$$

$$y^2 - y - 6 = 0$$

$$(y^2 - y - 6) = 0$$

$$(y^2 - 3y + 2y - 6) = 0$$

$$(y^2 - 3y) + (2y - 6) = 0$$

$$y(y-3) + 2(y-3) = 0$$

$$(y+2)(y-3) = 0$$

$y+2=0$ or $y-3=0$

$-2\ -2 \qquad +3\ +3$

$y=2$ OR $y=+3$

Check for any extraneous solutions:

$y = -2$

$-y = \sqrt{y+6}$

$-(-2) = \sqrt{-2+6}$

$+2 = \sqrt{4}$

$+2 = 2$

Yes

$y = 3$

$-(-3) = \sqrt{-3+6}$

$3 \neq \sqrt{3}$

* This is an extraneous solution & does not satisfy the original equation

* Review Problem 5 on pg. 635

* Got it #5

a) $6 - \sqrt{2x} = 10$

$-6 \qquad -6$

$-1(-\sqrt{2x} = 4)$

$(\sqrt{2x} = -4)^2$

* Already know there is no solution

$\frac{2x = 16}{2}$

$x = 8$

b) The principal root of a # is NEVER negative.

10.4 pg. 636 #8-48 even, skip 42

8) $\sqrt{t}+2=9$

$(\sqrt{t}=7)^2$

$t=49$

10) $\sqrt{n}-3=6$

$+3 \quad +3$

$(\sqrt{n}=9)^2$

$n=81$

12) $3-\sqrt{t}=-2$

$-3 \qquad -3$

$-1(-\sqrt{t}=-5)$

$(\sqrt{t}=5)^2$

$t=25$

14) $(\sqrt{10b+6}=6)^2$

$10b+6=36$

$-6 \quad -6$

$\frac{10b=30}{10}$

$b=3$

16) $(\sqrt{x-3}=4)^2$

$x-3=16$

$+3 \quad +3$

$x=19$

18) $S=\sqrt{\frac{A}{6}}$

$\left(9=\sqrt{\frac{A}{6}}\right)^2$

$\left(81=\frac{A}{6}\right)6$

$A=486\ cm^2$

20) $(\sqrt{2y}=\sqrt{9-y})^2$

$2y=9-y$

$+y \qquad +y$

$3y=9$

$y=3$

(22) $\left(\sqrt{s+10} = \sqrt{6-s}\right)^2$

$s+10 = 6-s$

$+s \qquad +s$

$\frac{2s = -4}{2}$

$s = -2$

(24) $\left(\sqrt{3m+1} = \sqrt{7m-9}\right)^2$

$3m+1 = 7m-9$

$+9 \qquad +9$

$3m+10 = 7m$

$\frac{10 = 4m}{4}$

$\frac{10}{4} = m$

$\frac{5}{2} = m$

or

2.5

(26) $\sqrt{12-n} = n$

$\sqrt{12+4} = -4$

$\sqrt{16} = -4$

$4 = -4$

$\sqrt{12-n} = n$

$\sqrt{12-3} = 3$

$\sqrt{9} = 3$

$3 = 3$ ✓

$-4$ is an extraneous solution

(28) $2a = \sqrt{4a+3}$

$2\left(\frac{3}{2}\right) = \sqrt{4\left(\frac{3}{2}\right)+3}$

$3 = \sqrt{6+3}$

$3 = \sqrt{9}$

$3 = 3$

✓

$2\left(-\frac{1}{2}\right) = \sqrt{4\left(-\frac{1}{2}\right)+3}$

$-1 = \sqrt{-2+3}$

$-1 = \sqrt{1}$

$-1 = 1$

$-\frac{1}{2}$ is an extraneous solution

30) $-t=\sqrt{-6t-5}$

$-(-5)=\sqrt{-6(-5)-5}$

$5=\sqrt{30-5}$

$5=\sqrt{25}$

$5=5$

✓

$-(-1)=\sqrt{-6(-1)-5}$

$1=\sqrt{6-5}$

$1=\sqrt{1}$

$1=1$

✓

No extraneous solutions

32) $(n=\sqrt{4n+5})^2$

$n^2=4n+5$

$n^2-4n-5=0$

$n^2-5n+1n-5=0$

$(n^2-5n)+(n-5)=0$

$n(n-5)+1(n-5)=0$

$(n+1)(n-5)=0$

$n=-1 \text{ or } 5$

ONLY 5 is a solution

-1 is the extraneous Solution

34) $(2y = \sqrt{5y+6})^2$

$4y^2 = 5y+6$

$4y^2 - 5y - 6 = 0$

$4y^2 - 8y + 3y - 6 = 0$

$4y(y-2) + 3(y-2) = 0$

$(4y+3)(y-2) = 0$

$4y+3=0$ | $y-2=0$

$-3 \quad -3$ | $+2 \quad +2$

$\frac{4y = -3}{4}$ | $y = 2$

$y = -\frac{3}{4}$

2 is the only solution

$-3/4$ is the extraneous solution

(36) $(\sqrt{d+12} = d)^2$

$d+12 = d^2$

$d^2 - d - 12 = 0$

$d^2 + 3d - 4d - 12 = 0$

$d(d+3) - 4(d+3) = 0$

$(d-4)(d+3) = 0$

$d = 4$ or $-3$

4 is the only solution

$-3$ is the extraneous solution

(38) $r = \sqrt{\frac{A}{4\pi}}$

$\left(6378 = \sqrt{\frac{A}{4\pi}}\right)^2$

$4\pi\left(40{,}678{,}884 = \frac{A}{4\pi}\right)$

$511{,}185{,}933\ Km^2$

(40) $r = \sqrt{\frac{V}{\pi h}}$

$\left(2 = \sqrt{\frac{75}{\pi h}}\right)^2$

$\frac{4\pi}{75}\left(4 = \frac{75}{\pi h}\right)$

$\frac{4\pi}{75} = \frac{1}{h}$

OR

$\frac{75}{4\pi} = h$

$h = 5.96$ or $6$ in

(44) $-6 - \sqrt{3y} = -3$

$+6 \qquad +6$

$\sqrt{3y} = 3$

No solution

(46) $\left(a = \sqrt{7a-6}\right)^2$

$a^2 = 7a - 6$

$a^2 - 7a + 6 = 0$

$a^2 - 6a - 1a + 6 = 0$

$a(a-6) - 1(a-6) = 0$

$(a-1)(a-6) = 0$

$a = 1$ or $6$

(48) $3 - \sqrt{4a+1} = 12$

$-3 \qquad -3$

$\left(-\sqrt{4a+1} = 9\right) -1$

No solution.

(50) $V = lwh$

$V = (10)(x)(x)$

A) $V = 10x^2$

B) $\frac{V = 10x^2}{10}$

$\sqrt{\frac{V}{10} = x^2}$

$x = \sqrt{\frac{V}{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}}$

$x = \frac{\sqrt{10V}}{10}$

C) Skip