

3.7 Absolute Value Equations & Inequalities

* Review

Absolute value * distance from zero

* $|A| = b$, therefore $A = b$ or $-b$

① $|3| = 3$

② $|-5| = 5$

③ $|3-5| = |-2| = 2$

④ $|-4| + |4| = 8$

⑤ $|x-7|$ when $x=3$, $|3-7| = |-4| = 4$

* To solve absolute value equations, isolate the absolute value expression.

⑥ $|y| = 17$

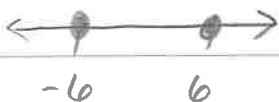
$y = \pm 17$



⑦ $|y-5| = 1$

$|y| = 6$

$y = \pm 6$

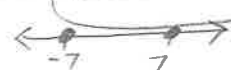


⑧ $2|x| + 1 = 15$

$2|x| = 14$

$|x| = 7$

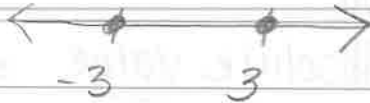
$x = \pm 7$



* Review Problem #1 on pg. 207

* Got it? #1)

$$\begin{array}{r} |n| - 5 = -2 \\ +5 \quad +5 \end{array}$$



$$|n| = 3$$

$$n = \pm 3 \text{ or } \{-3, 3\}$$

* Addt'l Examples

⑨ $|m-4|=1$ * set up the equation 2 ways



$$m-4=1$$

$$+4 \quad +4$$

$$m=5$$

$$m-4=-1$$

$$+5 \quad +5$$

$$m=4$$

$$m=4, 5 \text{ or } \{4, 5\}$$

⑩ $-3|m| + 5|m| - 3 = 1$ * combine like terms

$$2|m| - 3 = 1$$

$$+3 \quad +3$$

$$2|m| = 4$$

$$\underline{\quad}$$

$$|m| = 2$$

$$m = 2 \text{ or } -2 \text{ or } \{-2, 2\}$$

$$(11) |2a+1| = 5$$

$$2a+1=5$$

$$\begin{array}{r} -1 \\ -1 \end{array}$$

$$\underline{2a=4}$$

$$2$$

$$a=2$$

$$2a+1=-5$$

$$\begin{array}{r} -1 \\ -1 \end{array}$$

$$\underline{2a=-6}$$

$$2$$

$$a=-3$$

$$\boxed{a=-3, 2} \text{ or } \{3, 2\}$$

* Review Problem 3 on pg. 208

* Got it? #3

$$3) |3x-6|-5 = -7$$

$$\begin{array}{r} +5 \\ +5 \end{array}$$

$$|3x-6| = -2$$

No solution, the absolute value of an expression cannot equal a negative number

* Review Problem 2 on pg. 208

* Got it? #2

$$2) d = |80 - 5t|$$

80 = 80 feet away (friends starting distance)
5 = speed of ft/seconds



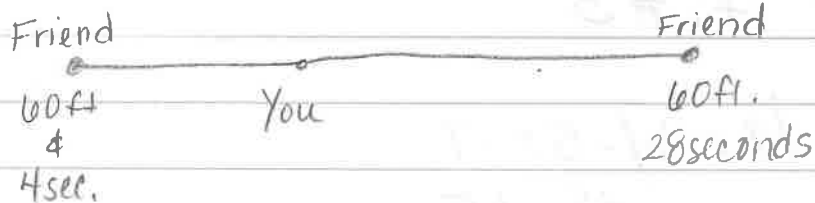
$$60 = |80 - 5t|$$

$$\begin{array}{r} 60 = 80 - 5t \\ -80 \quad -80 \\ \hline -20 = -5t \\ -5 \end{array} \qquad \begin{array}{r} -60 = 80 - 5t \\ -80 \quad -80 \\ \hline -140 = -5t \\ -5 \end{array}$$

$$t = 4$$

$$t = 28$$

* Your friend is 60 feet away at 4 seconds and 28 seconds.



* Inequalities with Absolute value are compound inequalities

* $<$ or \leq is an "and" compound inequality

* $>$ or \geq is an "or" compound inequality

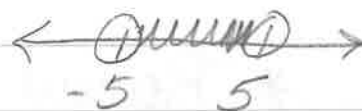
(Think of "briator" instead of brreater)

Examples

(12) $|3x| < 15$

$$\begin{array}{r} 3x < 15 \\ \hline 3 \end{array}$$

$$\begin{array}{r} 3x > -15 \\ \hline 3 \end{array}$$



$x < 5$ and $x > -5$

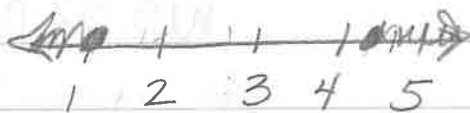
$-5 < x < 5$

* When you write the problem with your answer as a negative, you must flip the sign.

(13) $|3x-8| \geq 5$

$$\begin{array}{r} 3x-8 \geq 5 \\ +8 \quad +8 \\ \hline 3x \geq 13 \\ \hline 3 \end{array}$$

$$\begin{array}{r} 3x-8 \leq -5 \\ +8 \quad +8 \\ \hline 3x \leq 3 \\ \hline 3 \end{array}$$

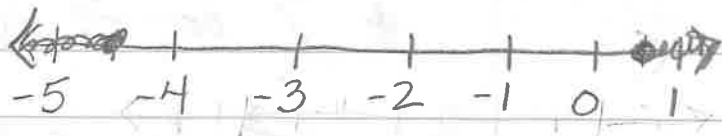


$x \geq 4\frac{1}{3}$ or $x \leq 1$

* Review Problem 4 on pg. 209

* Got it? #4

$$|2x+4| \geq 5$$



$$2x+4 \geq 5$$
$$\begin{array}{r} -4 \\ -4 \end{array}$$

$$2x+4 \leq -5$$
$$\begin{array}{r} -4 \\ -4 \end{array}$$

$$2x \geq 1$$

$$2x \leq -9$$

2

2

$$x \geq \frac{1}{2} \text{ or } x \leq -4\frac{1}{2}$$

* Review Problem 5 on pg. 210

* Got it? #5

A)

$$|w-32| \leq 0.05$$



$$w-32 \leq 0.05$$

$$w-32 \geq -0.05$$

$$\begin{array}{r} +32 \\ +32 \end{array}$$

$$\begin{array}{r} +32 \\ +32 \end{array}$$

$$w \leq 32.05$$

$$w \geq 31.95$$

$$31.95 \leq w \leq 32.05$$


B)

No, 213 is part of the absolute value expression. You need to write the absolute value inequality as a compound inequality first.

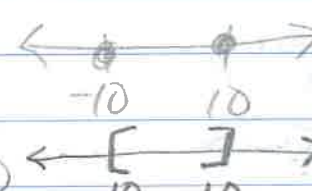
Equations #10-30 even
#50-56 even & #64

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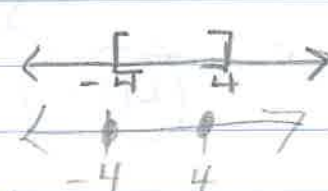
Inequalities #32-46 even
58-60 even

(10) $4 = |y|$
 $y = 4, -4$


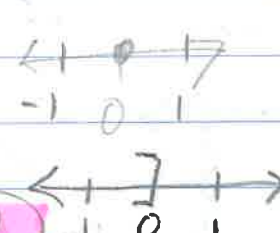
(22) $-3|2w| = -12$
 $|2w| = 4$

(12) $7 = |s| - 3$
 $+3 \quad +3$
 $10 = |s|$
 $s = 10, -10$


$2w = 4 \quad 2w = -4$
 $2 \quad 2$
 $w = 2, -2$

(14) $5|d| = 20$
 $|d| = 4$
 $d = -4, 4$


(24) $2|d+4| = 8$
 $d+4 = 4 \quad d+4 = -4$
 $d = 0 \quad d = -8$
 $d = 0, -8$

(16) $|y| + 3 = 3$
 $-3 \quad -3$
 $|y| = 0$
 $y = 0$


(26) $|3t - 2| + 6 = 2$
 $-6 \quad -6$
 $|3t - 2| = -4$
 no solution

(18) $|c+4| = 6$
 $c+4 = 6 \quad c+4 = -6$
 $4 \quad 4$
 $c = 2, -10$

(18) $3|x+2| + 4 = 13$
 $-4 \quad -4$

(20) $3 = |m+2|$
 $3 = m+2 \quad -3 = m+2$
 $-2 \quad -2$
 $m = 1, -5$

$3|x+2| = 9$
 $|x+2| = 3 \quad x+2 = -3$
 $x = 1 \quad x = -5$
 $x = -5, 1$

$$(30) \quad |-3n| - 2 = 4$$

$$\begin{aligned} &+2 \quad +2 \\ &|-3n| = 6 \\ &-3n = 6 \quad -3n = -6 \\ &\quad -3 \quad \quad -3 \end{aligned}$$

$$n = -2, 2$$

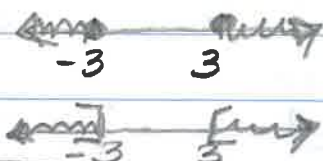
$$(38) \quad |2c - 5| < 9$$

$$\begin{aligned} 2c - 5 < 9 & \quad \& \quad 2c - 5 > -9 \\ +5 \quad +5 & \quad \quad +5 \quad +5 \\ \underline{2c < 14} & \quad \quad \underline{2c > -4} \\ 2 & \quad \quad \quad 2 \end{aligned}$$

$$c < 7 \quad \& \quad c > -2$$

$$-2 < c < 7$$

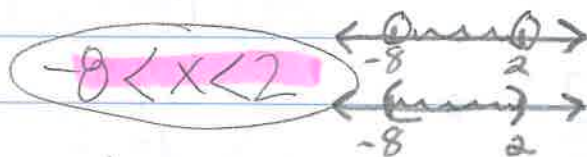
$$(32) \quad |x| \geq 3$$



$$x \geq 3 \text{ or } x \leq -3$$

$$(34) \quad |x + 3| < 5$$

$$\begin{aligned} x + 3 < 5 & \text{ and } x + 3 > -5 \\ -3 \quad -3 & \quad \quad -3 \quad -3 \\ x < 2 & \text{ and } x > -8 \end{aligned}$$



$$-8 < x < 2$$

$$(40) \quad |4w + 1| > 11$$

$$\begin{aligned} 4w + 1 > 11 & \text{ or } 4w + 1 < -11 \\ -1 \quad -1 & \quad \quad -1 \quad -1 \\ \underline{4w > 10} & \quad \quad \underline{4w < -12} \\ 4 & \quad \quad \quad 4 \end{aligned}$$

$$w > \frac{5}{2} \text{ or } w < -3$$

$$(36) \quad |y - 2| \leq 1$$

$$\begin{aligned} y - 2 \leq 1 & \text{ and } y - 2 \geq -1 \\ +2 \quad +2 & \quad \quad +2 \quad +2 \\ y \leq 3 & \text{ and } y \geq 1 \end{aligned}$$

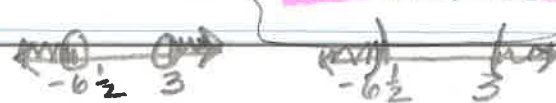
$$1 \leq y \leq 3$$



$$(42) \quad |4x + 7| > 19$$

$$\begin{aligned} 4x + 7 > 19 & \text{ or } 4x + 7 < -19 \\ -7 \quad -7 & \quad \quad -7 \quad -7 \\ \underline{4x > 12} & \quad \quad \underline{4x < -26} \\ 4 & \quad \quad \quad 4 \end{aligned}$$

$$x > 3 \text{ or } x < -6\frac{1}{2}$$



$$(44) |3d-7| > 28$$

$$3d-7 > 28 \quad \text{or} \quad 3d-7 < -28$$

$$+7 \quad +7$$

$$+7 \quad +7$$

$$3d > 35$$

$$3d < -21$$

$$3$$

$$3$$

$$d > 11\frac{2}{3} \quad \text{or} \quad d < -7$$



$$(46) |5m-9| \geq 24$$

$$5m-9 \geq 24 \quad \text{or} \quad 5m-9 \leq -24$$

$$+9 \quad +9$$

$$+9 \quad +9$$

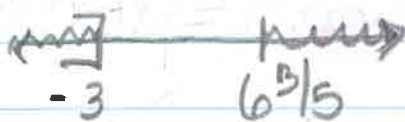
$$5m \geq 33$$

$$5m \leq -15$$

$$5$$

$$5$$

$$m \geq 6\frac{3}{5} \quad \text{or} \quad m \leq -3$$



$$(52) |f| - \frac{2}{3} = \frac{5}{6}$$

$$+ \frac{2}{3} \quad + \frac{2}{3} = \frac{4}{6}$$

$$|f| = \frac{9}{6}$$

$$f = \frac{9}{6} \quad \text{or} \quad -\frac{9}{6}$$

$$(54) |t| + 2.7 = 4.5$$

$$-2.7 \quad -2.7$$

$$|t| = 1.8$$

$$t = \pm 1.8$$

$$(56) \left(\frac{|y|}{-3} = 5 \right) - 3$$

No solution

$$|y| = 15$$

$$y = \pm 15$$

$$(57) |x| = 6$$

$$x = 6 \quad \text{or} \quad x = -6$$

$$5/20 = 3$$

$$|c+7| > \frac{7}{8}$$

$$(58) \quad \frac{7}{8} < |c+7|$$

$$\begin{array}{ccc} \frac{7}{8} < c+7 & \text{or} & -\frac{7}{8} > c+7 \\ -7 & -7 & +7 & -7 \\ -6\frac{1}{8} < c & & -7\frac{7}{8} > c \end{array}$$

$$c > -6\frac{1}{8} \text{ or } c < -7\frac{7}{8}$$

$$(60) \quad |-3d| \geq 6.3$$

$$\begin{array}{ccc} -3d \geq 6.3 & \text{or} & -3d \leq -6.3 \\ -3 & & -3 \end{array}$$

$$\begin{array}{ccc} *FLIP & d \geq -2.1 & d \leq 2.1 \quad *FLIP \\ & d \leq -2.1 & \text{or } d \geq 2.1 \end{array}$$

(64) No solution, cannot have a negative absolute value expression

$$(72) \quad |x+4| = 3x$$

$$\begin{array}{ccc} x+4 = 3x & & x+4 = -(3x) \\ -x & -x & -x & +x \\ 4 = 2x & & 4 = -4x \\ 2 = x & & -1 = x \end{array}$$